

Gravitational Waves: Models and Experiments on Waveforms, Effects and Detection

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In this contribution we present models and experiments that are designed to convey an idea of the nature of gravitational waves to students. These teaching materials have been developed for the “Schülerlabor Raumzeitwerkstatt” (student lab on relativistic physics) at Hildesheim university, where they are used for teaching students in grades 9 to 13 (aged 15 to 19 years).

Introduction

Albert Einstein presented the general theory of relativity in 1915, and deduced a wave solution as early as 1916. From 1958 onwards searches for gravitational waves were conducted, initially with resonance detectors and from 2002 onwards with laser interferometers. The existence of gravitational waves was first confirmed by observations of the Hulse-Taylor binary pulsar. This pulsar, discovered in 1974, is in orbit around another neutron star. The orbital period of the binary system decreases as the orbit shrinks due to the emission of gravitational waves that carry away energy. The measured decrease in orbital period is in excellent agreement with the prediction of general relativity for the energy loss due to gravitational waves. The first direct detection took place in September 2015 when the two LIGO interferometers (USA) measured the signal of two coalescing black holes (Abbott et al. 2016). Gravitational waves, like electromagnetic waves, propagate at the speed of light. Their sources are accelerated massive bodies. For gravitational waves to arise, the accelerated motion must have a certain asymmetry. A typical example is the motion of stars orbiting in a binary system. A body oscillating back and forth, on the contrary, would not produce gravitational waves, neither would a spherically symmetric collapse. The interaction of gravitational waves with matter is very weak and their detection therefore extremely difficult.

In this contribution we present models and experiments that have been developed for the “Schülerlabor Raumzeitwerkstatt” (student lab on relativistic physics) at Hildesheim university. The student lab provides an introduction to different topics of the special and the general theory of relativity, gravitational waves being one of these topics. The participants are high school students in grades 9 to 13 (aged 15 to 19 years).

In order to teach a topic in the field of general relativity to high school students, one must find approaches that require no more than elementary mathematics. In the case of gravitational waves this turns out to be difficult because simple approaches that proved their worth in other cases are not applicable here: The phenomenon cannot be inferred by thought experiments based on the equivalence principle the way it is done for gravitational light deflection and gravitational redshift. Neither can one give a description using the terms and definitions of classical physics as is sometimes done in

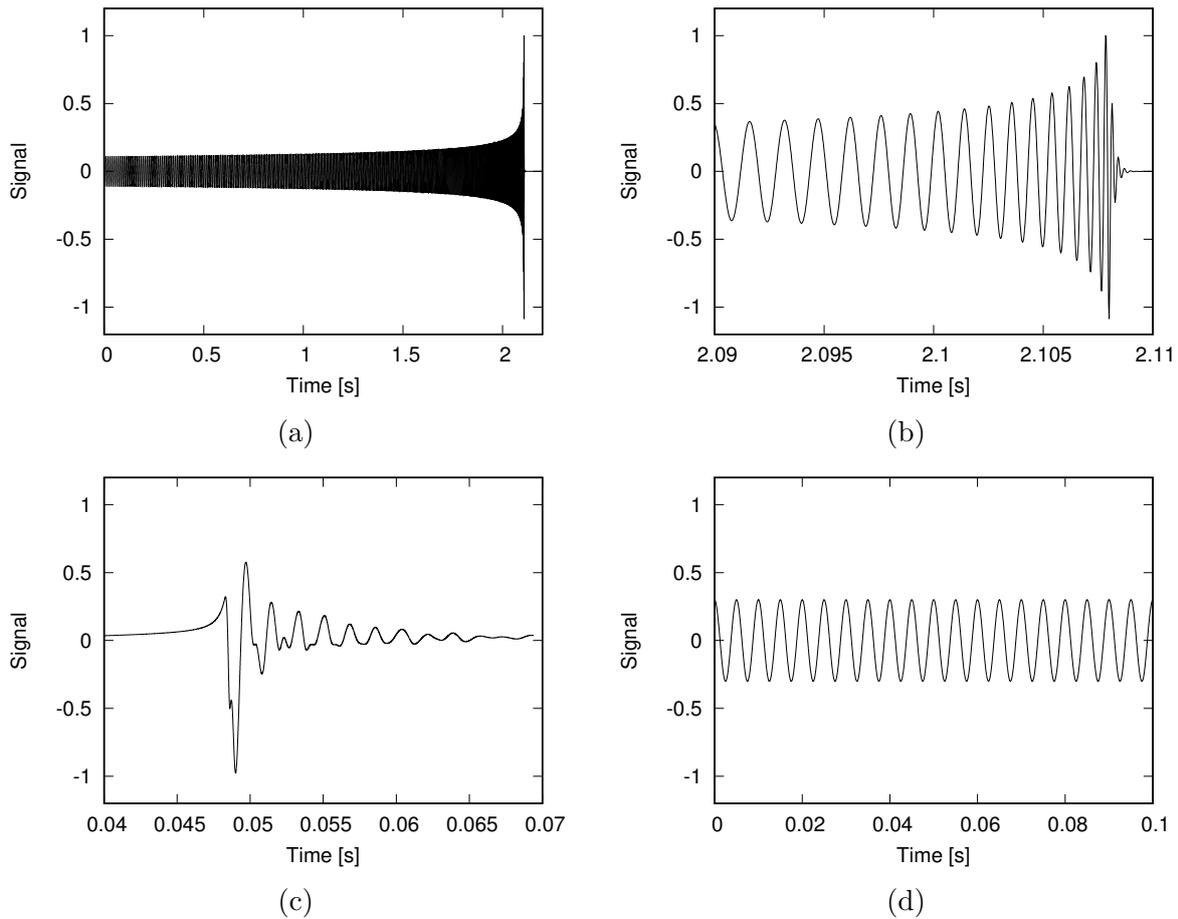


Figure 1: Typical gravitational waveforms expected from coalescing neutron stars (a, detail: b), supernovae (c) and pulsars (d). The data shown in part (c) are from Dimmelmeier et al. 2002.

the case of black holes. In fact, the Newtonian theory of gravity being static gravitational waves do not have a classical counterpart.

The models and experiments described below are meant to convey a notion of the nature of gravitational waves and to point out the order of magnitude of some key quantities. The phenomenon is approached from three different points of view: What kind of signal does one expect and from which sources? What is the impact of a gravitational wave on matter? How are gravitational waves detected?

Waveforms and Sources

In this section three types of sources are presented together with the waveforms of their gravitational waves: Coalescing neutron stars, supernovae, and pulsars.

A binary system consisting of two neutron stars is a gravitational wave source due to the orbital motion of the stars. As a result of the associated energy loss the stars

gradually get closer and the orbital frequency increases. An estimated value of the orbital frequency just before coalescence can be obtained using the Newtonian formula for the orbital frequency of a system of two equal point masses M a distance D apart:

$$f = \frac{1}{2\pi} \sqrt{\frac{2GM}{D^3}}.$$

With the distance D set to two neutron star radii and with typical values of $M = 1.4$ solar masses and $D = 20$ km, the resulting orbital frequency is $f = 1100$ Hz. The gravitational waves emitted by the binary have twice this frequency as can be seen by the following argument: After half a revolution, the two stars have just swapped their positions. The second half of the orbit therefore repeats the motion pattern of the first half, and the gravitational wave signal is repeated accordingly. One orbital period therefore comprises two periods of the gravitational wave signal. Therefore, a frequency of about 2200 Hz is to be expected just before coalescence. The amplitude of the emitted gravitational wave gradually rises while the stars approach, and reaches a sharp peak at coalescence. After coalescence, when a black hole has formed, the signal shows a few additional oscillations with smaller amplitude. This is the so-called ringdown phase (Fig. 1a, b). The wave form shown in Fig. 1a, b is made up of two approximate solutions for times before and after coalescence, respectively. The signal during the coalescence phase must be computed with a numerical simulation. This comparatively short section of the signal is not described realistically in Fig. 1.

In a supernova explosion, gravitational waves are produced when the core of the star collapses to form a neutron star or a black hole. For gravitational waves to be emitted, the collapse must have deviations from spherical symmetry. The resulting signal has an abrupt beginning and a short high peak. The signal shown in Fig. 1c is the result of numerical simulations (Dimmelmeier et al. 2002).

Pulsars are rotating neutron stars. Pulsar frequencies have been observed in the range of Herz to Kilohertz. To give rise to gravitational waves the rotating neutron star must have a shape that is not perfectly symmetric with respect to its axis of rotation, a possible cause for such an asymmetry being a deformation of the crust. The emission of gravitational waves gradually slows down the rotation of the pulsar. The gravitational wave signal of the pulsar is long-term stable. Fig. 1d shows a pulsar signal with a frequency of 200 Hz.

Generally speaking the frequency of a gravitational wave is of the order of magnitude of the frequency of the periodic motion in the source. For periodic motion in an astronomical object, the frequency cannot be arbitrarily high; expected values are about 10 kHz maximum. This means that the frequencies of the expected gravitational waves in many cases lie in the range between 20 Hz and 20 000 Hz in which the human ear can detect sound waves. Thus, by generating a sound wave with a given waveform, one may listen to a gravitational wave signal. Sound files for the wave forms shown in Fig. 1 are available from www.spacetime.travel.org as supplement to this contribution. The sound files were created using the open source tool scilab.

The visitors of the student lab “Raumzeitwerkstatt” at Hildesheim university get to know sources and wave forms by studying the four aspects that were described above: Every



Figure 2: Students match cards with pieces of information belonging to the same type of source.

type of source is introduced by a short description, an image (observed or simulated), a diagram of a typical waveform and a sound file that renders the wave form as an acoustic signal. Students are given texts, images and diagrams on separate cards as well as the sound files. Their task is to match cards and sound files belonging to the same type of source. Texts, graphs and images for the cards are available ready for printing from www.spacetime.travel.org as supplement to this contribution (Fig. 2).

Impact of a gravitational wave on matter

Gravitational waves are produced by masses in accelerated motion, they propagate into space and they have a gravitational effect far away from their sources.

An electromagnetic wave becomes noticeable by accelerating charged particles. The acceleration depends on the ratio of charge to mass and is, therefore, different for different particles. The acceleration of charged particles relative to uncharged ones betrays the presence of an electromagnetic field.

One may ask if a gravitational wave can be identified in a similar manner by the acceleration of massive particles. This, however, is not the case and at this point the analogy to electromagnetic waves fails. The gravitational force F acting on a particle with mass m produces the acceleration $a = F/m$. Since the gravitational force F is proportional to m , the acceleration is independent of the particle mass. Therefore, in a gravitational field, unlike in the electromagnetic case, all particles undergo the same acceleration (at the same position). There is no relative acceleration between particles that would betray the presence of the gravitational field. For relative acceleration to occur, particles must

be at different locations with different gravitational fields. One speaks of tidal forces named after the tides that are generated in this way by the inhomogeneous gravitational field of the moon. In summary, the characteristic feature of a gravitational wave is a time-varying tidal force. To detect a gravitational wave means to detect tidal forces.

The above argument is formulated within the Newtonian theory of gravity. However, the mass-independence of gravitational acceleration together with the implication that relative accelerations are the result of tidal forces is also part of the general theory of relativity so that the argument is also valid in this framework.

The following two models illustrate the tidal effects of a gravitational wave. We imagine a set of 16 particles that are in free fall (i. e. that experience no non-gravitational forces), for instance freely floating in space. The particles are arranged in a square grid (Fig. 3a). Now we image a gravitational wave that propagates in the direction perpendicular to the plane of the grid. We take it to be a plane wave, because far away from the source a small section of the wave front is virtually plane. And, finally, we consider it to be monochromatic.

The action of the gravitational wave is to change the distances of the particles with respect to each other (Fig. 3b-d). The grid remains planar in the process, i. e. the action of the wave is within the plane perpendicular to its propagation. This means that gravitational waves are transverse waves. The acrylic model (Fig. 3e) illustrates this effect of the gravitational wave. It is made up of 13 transparent plates of acrylic glass that are lined up in a wooden rack. On each plate, dots mark the arrangement of the particles at a certain instant of time. The model covers one complete period with an interval of 1/12 of the wave period between neighbouring plates.

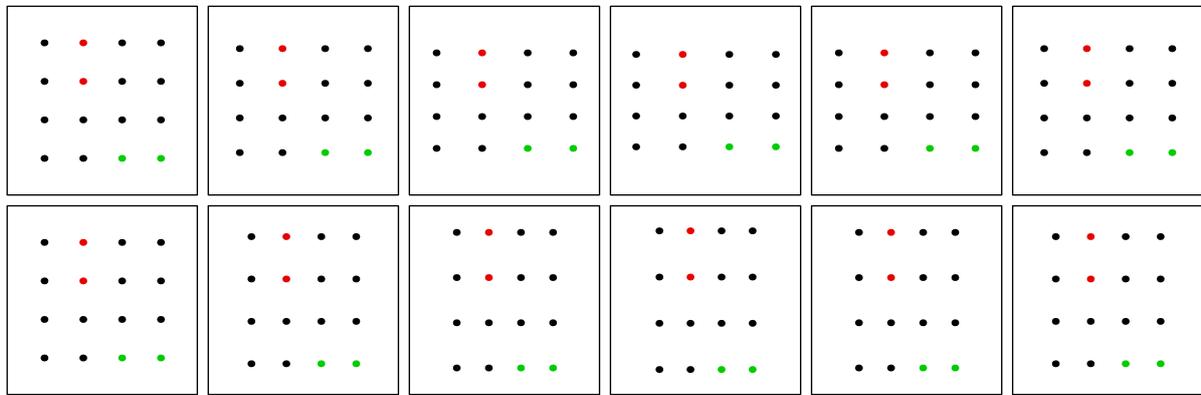
The plates illustrate the pattern of distance changes in the gravitational wave: When distances increase in the horizontal direction, they simultaneously decrease in the vertical direction and vice versa. In the model the plates have 12 cm edge length, the dots of the symmetric grid are $d_0 = 2$ cm apart in both the horizontal and the vertical direction, and over one period the distances vary between 1.6 cm und 2.4 cm according to

$$d_x = d_0(1 + a \sin(2\pi ft)), \quad d_y = d_0(1 - a \sin(2\pi ft))$$

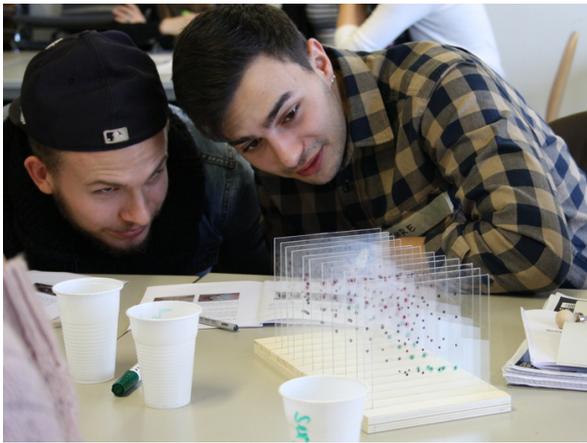
with $a = 0.2$. The resulting pattern is clearly visible when peering through the row of plates. The pattern is further emphasized by the use of colours: Two particles that lie side by side are marked green on all plates, two others that are one above the other are marked red. Peering through the plates one can see that the distance changes of these two pairs are always opposite to each other.

The amplitude of a gravitational wave is defined via the changes in distance of neighbouring particles: When the distance is l and the gravitational waves changes it by a maximum of δl , then the wave amplitude is defined to be $h = 2(\delta l/l)$. For the acrylic model the amplitude is $2\delta l/l = 2d_0a/d_0 = 0,4$. This is useful for the purpose of illustration but far larger than the amplitudes that one expects to measure. The maximum amplitude of the strong signal that was detected in September 2015 is 10^{-21} (Abbott et al. 2016).

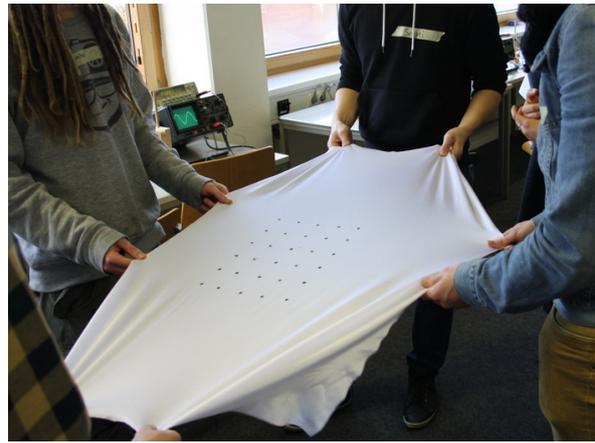
Another demonstration of the effects of gravitational waves uses an elastic fabric. Again, 16 particle positions on a square grid are marked by dots. Students that have examined



(a)



(b)



(c)

Figure 3: The effect of a plane monochromatic gravitational wave on a set of free particles, illustrated by two models. Free particles are arranged in a square grid (a). A gravitational wave then propagates in a direction perpendicular to the grid. While the particles are inside the wave their distances vary with time (a, from left to right). The acrylic plates show the arrangement of the particles at time steps that $1/12$ wave period apart (b). The time varying particle pattern can be simulated with the help of an elastic fabric (c).

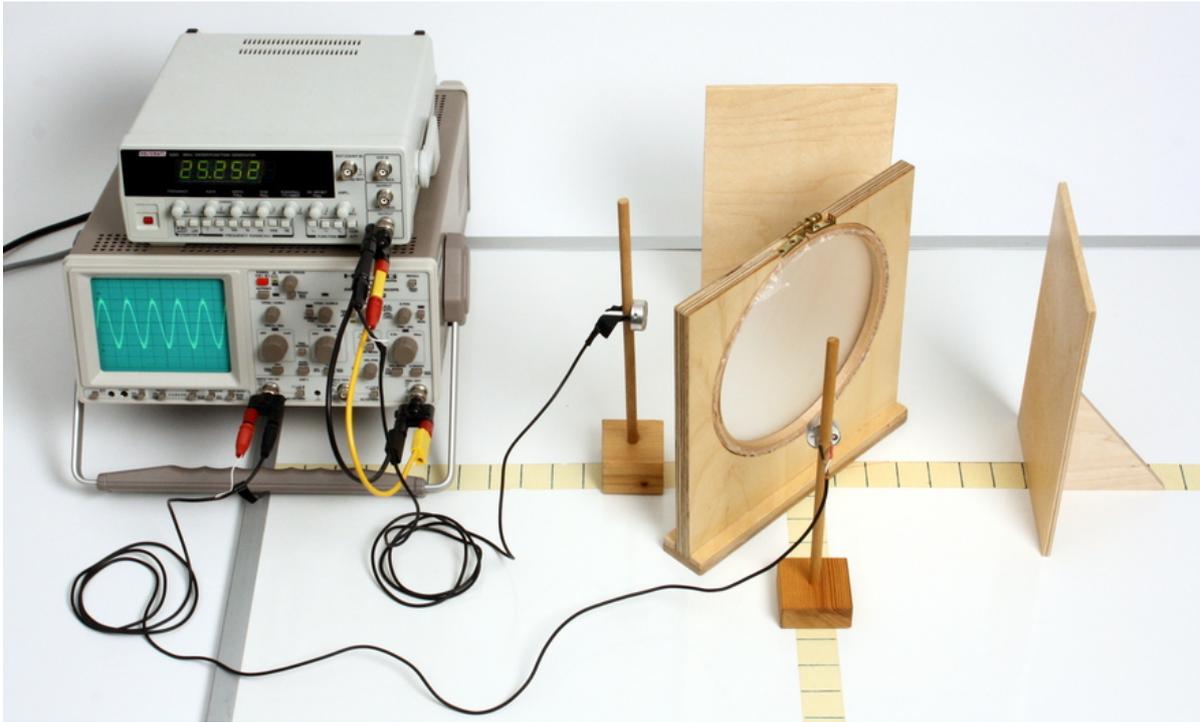


Figure 4: A Michelson interferometer working with ultrasound.

the acrylic model, are then asked to simulate the pattern of distance changes with the fabric (Fig. 3f). In order to reproduce the variation with time as shown in the acrylic model, four persons must each grab one edge of the fabric and must stretch it by turns in the two directions.

Both models are easily built. For the acrylic model, acrylic glass of 1.5 mm thickness was cut into plates with 12 cm edge length. The elastic fabric is of the type that is usually used for swimwear and is available in draperies.

Detection of gravitational waves

A number of gravitational wave detectors is being operated or being built in several different countries; together they form a global network. These detectors are laser interferometers, similar to a Michelson interferometer. They are designed to detect the tiny distance changes caused by gravitational waves. In Germany, there is the German-British detector GEO 600 with an armlength of 600 metres located near Hanover. Two interferometers with 4 kilometres armlength each are situated in the USA in Livingston (Louisiana) and Hanford (Washington), respectively. The first detection of a gravitational wave signal in September 2015 was achieved with these two detectors. The signal originates from the coalescence of two black holes and was seen in both LIGO detectors. In order to explain the basic principle of the measurement to students, we use a Michelson interferometer operating with ultrasound. At 25 kHz frequency or 1.4 cm ultrasonic wavelength it is easy to manually change the armlength by a fraction of the wavelength.

Fig. 4 shows how to assemble the ultrasound interferometer: A sender is connected to a frequency synthesiser. A “semi-transparent mirror” built with a plastic foil, splits the beam into two parts. Each partial beam passes along one arm of the interferometer and is reflected at its end by a wooden board, acting as “mirror”. The two beams are made to interfere at the position of the detector, and the detected signal is displayed on the oscilloscope. Students first adjust the armlengths for destructive interference, i. e. for the smallest possible signal at the detector. They then change one of the armlengths and observe that this can be detected by a change in the signal.

The ultrasound interferometer has been built from parts and materials that are easily available. The ultrasound senders (ultrasound transducer RD25K2, 25 kHz, available e. g. from Pollin Electronic) are attached to wooden supports. The supports have magnets inserted in their bases that help to position them on a magnet board. Wooden slats are used as mirrors at the ends of the two arms. For the semi-transparent mirror, different types of material were tested in order to obtain transmitted and reflected beams of comparable intensity. In our interferometer, a piece of plastic foil from a Yellow Bag is used which meets the requirements quite well. Quivering of the plastic foil in air drafts is suppressed by firmly fixing it in an embroidery hoop that in turn is mounted on a wooden support.

References

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